$\boxed{S_{III}}$

1) $f:(1,+\infty)\to\mathbb{R},\ f(x)=\dfrac{x^2}{x-1}$

a) $\lim\limits_{x\to1}\dfrac{x^2}{x-1}=\lim\limits_{\substack{x\to1\\x>1}}\dfrac{x^2}{x-1}=\dfrac{1}{0_+}=+\infty \Rightarrow \underline{x=1\ \text{A.V}}$

b) $\lim\limits_{x\to2}\dfrac{f(x)-4}{x-2}=\lim\limits_{x\to2}\dfrac{\frac{x^2}{x-1}-4}{x-2}=\lim\limits_{x\to2}\dfrac{\frac{x^2-4x+4}{x-1}}{x-2}=$

$=\lim\limits_{x\to2}\dfrac{x^2-4x+4}{(x-1)\cdot(x-2)}=\lim\limits_{x\to2}\dfrac{(x-2)^{\cancel{2}}}{(x-1)\cdot\cancel{(x-2)}}=\lim\limits_{x\to2}\dfrac{x-2}{x-1}=\dfrac{2-2}{2-1}=\underline{0}$

c) $\lim\limits_{x\to+\infty}\dfrac{x^2}{x-1}=+\infty$ (grad Numărător (2) > grad numitor (1))

$\Rightarrow$ nu are A. oriz.

$y=mx+n$

$m=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x^2-x}=1$

$n=\lim\limits_{x\to+\infty}f(x)-m\cdot x=\lim\limits_{x\to+\infty}\dfrac{x^2}{x-1}-x=\lim\limits_{x\to+\infty}\dfrac{\cancel{x^2}-\cancel{x^2}+x}{x-1}=\underline{1}$

$\boxed{y=1\cdot x+1}$ asimpt. oblică ✓

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2) $f(x)=\begin{cases}e^{x+1}-3, & x\le-1\\ 2x^3+(a-3)x-4, & x>-1.\end{cases}$

a) $l_s(-1)=l_d(-1)\Leftrightarrow e^0-3=-2+(a-3)\cdot(-1)-4\Rightarrow 1-3=-2-a+3-4\Rightarrow$
$-2=-2-a-1\Rightarrow\boxed{a=-1}$

b) $f(x)+2\le0\ \forall x\le-1 \Leftrightarrow e^{x+1}-3+2\le0\Leftrightarrow e^{x+1}-1\le0$
$e^{x+1}\le1\Rightarrow e^{x+1}\le e^0$
$x+1\le0\Rightarrow x\le-1$ (A)

c) $a=-1$; $f(x)=0$ are sol. pe $[0,2]$
$f(x)=2x^3-4x-4.$
$f(0)=-4$; $f(2)=2\cdot8-4\cdot2-4=16-8-4=4$
$f(0)\cdot f(2)=-16<0$
f cont.
$\Rightarrow f$ are cel puțin o soluție pe $[0;2]$.