

II

$$2. \quad x * y = xy - 5x - 5y + 30 \quad (1)$$

$$a) \quad x * y = (x-5)(y-5) + 5 = xy - 5x - 5y + 25 + 5 = (1)$$

$$b) \quad x * x^{-1} = e$$

$$x * e = x \Leftrightarrow xe - 5x - 5e + 30 = x$$

$$e(x-5) = x + 5x - 30$$

$$e(x-5) = 6x - 30 \Rightarrow e(x-5) = 6(x-5)$$

$$\Rightarrow \underline{e = 6}$$

$$x * x^{-1} = 6 \Rightarrow x \cdot x^{-1} - 5x - 5x^{-1} + 30 = 6$$

$$x^{-1}(x-5) = 5x - 24 \Rightarrow x^{-1} = \frac{5x-24}{x-5}$$

$$x^{-1} = \frac{5x-24-1+1}{x-5} = \frac{5x-25+1}{x-5} = \frac{5(x-5)+1}{x-5} = 5 + \frac{1}{x-5} \in \mathbb{Z}$$

$$x-5 \in \mathcal{D}_1 = \{\pm 1\} \Rightarrow x-5=1 \Rightarrow \underline{x=6} ; x-5=-1 \Rightarrow \underline{x=4}$$

c)  $d_1 * d_2 * \dots * d_p$ ,  $d_1, d_2, \dots, d_p$  divizorii naturali ai lui 2015.

$$x * y = (x-5)(y-5) + 5.$$

$$x * 5 = \frac{(x-5)(5-5) + 5}{0} = 5 \quad x * 5 = 5 \quad \forall x \in \mathbb{Z}$$

$$5\text{-divizor pt. 2015} \Rightarrow 5 * \underbrace{(d_1 * d_2 * \dots * d_p)}_x = 5, \quad \forall x \in \mathbb{Z}$$

, \* asociativă !