$\boxed{S_{III}}$

1. $f:(-1;+\infty)\to\mathbb{R},\ f(x)=x-\ln(x+1)$.

a) $f'(x) = 1-\frac{1}{x+1} = \frac{x+1-1}{x+1} = \frac{x}{x+1}$

b) $\lim\limits_{x\to 1}\frac{x-f(x)-\ln 2}{x-1} = \frac{\cancel{x}-\cancel{x}+\ln(x+1)-\ln 2}{x-1} = \lim\limits_{x\to 1}\frac{\ln\left(\frac{x+1}{2}\right)}{x-1} = \left(\frac{0}{0}\right)$

$\lim\limits_{x\to 1}\frac{\ln\left(1+\frac{x+1}{2}-1\right)}{x-1} = \lim\limits_{x\to 1}\underbrace{\frac{\ln\left(1+\frac{x-1}{2}\right)}{\frac{x-1}{2}}}_{\to 1}\cdot\frac{1}{2} = 1\cdot\frac{1}{2} = \frac{1}{2}$.

c) $\ln(x+1)\le x \Rightarrow 0\le x-\ln(x+1) \Rightarrow f(x)\ge 0$.

$f'(x) = 1-\frac{1}{x+1},\ x\in(-1;+\infty)$

$f'(0) = 0$

x	-1		0		$+\infty$
$f'(x)$		$-\ -\ -$	0	$+\ +\ +$	
$f(x)$		$\searrow$	$f(0)$	$\nearrow$ $f(x)$	

$\Rightarrow f(x)\ge f(0)$

$\Rightarrow f(x)\ge 0$

2. $f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{x}{x^2+1}$.

a) $\int_0^1 f(x)\,dx = \frac{1}{2}\int_0^1\frac{2x}{x^2+1}\,dx = \frac{1}{2}\int_1^2\frac{dt}{t} = \frac{1}{2}\ln t\Big|_1^2 = \frac{1}{2}(\ln 2-\underbrace{\ln 1}_{0})$

$= \frac{1}{2}\ln 2$.

$(x^2+1=t)' \Rightarrow 2x\,dx = dt$

$x=0\Rightarrow t=1$

$x=1\Rightarrow t=2$