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1. $f: (-1; +\infty) \rightarrow \mathbb{R}, f(x) = x - \ln(x+1).$

a) $f'(x) = 1 - \frac{1}{x+1} = \frac{x+1-1}{x+1} = \frac{x}{x+1}$

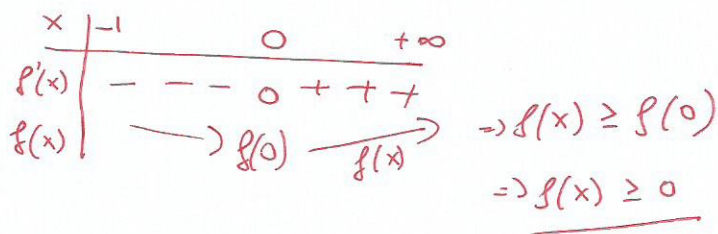
b) $\lim_{x \rightarrow 1} \frac{x - f(x) - \ln 2}{x-1} = \frac{x - x + \ln(x+1) - \ln 2}{x-1} \stackrel{0}{=} \lim_{x \rightarrow 1} \frac{\ln\left(\frac{x+1}{2}\right)}{x-1} = \left(\frac{0}{0}\right)$

$\lim_{x \rightarrow 1} \frac{\ln\left(1 + \frac{x+1}{2} - 1\right)}{x-1} = \lim_{x \rightarrow 1} \frac{\ln\left(1 + \frac{x-1}{2}\right)}{\frac{x-1}{2}} \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}.$

c) $\ln(x+1) \leq x \Rightarrow 0 \leq x - \ln(x+1) \Rightarrow f(x) \geq 0.$

$f'(x) = 1 - \frac{1}{x+1}, x \in (-1; +\infty)$

$f'(0) = 0$



2. $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{x}{x^2+1}.$

a) $\int_0^1 f(x) dx = \frac{1}{2} \int_0^1 \frac{2x}{x^2+1} dx = \frac{1}{2} \int_1^2 \frac{dt}{t} = \frac{1}{2} \ln t \Big|_1^2 = \frac{1}{2} (\ln 2 - \ln 1) = \frac{1}{2} \ln 2.$

$(x^2+1=t) \Rightarrow 2x dx = dt$

$x=0 \Rightarrow t=1$

$x=1 \Rightarrow t=2$