2. b) $\displaystyle\int_0^1 \frac{f(x) + x^2 \cdot f(x)}{x^4+1}\,dx = \frac{\pi}{8}.$

$$\int_0^1 \frac{\frac{x}{x^2+1} + x^2\cdot\frac{x}{x^2+1}}{x^4+1}\,dx = \int_0^1 \frac{\frac{x+x^3}{x^2+1}}{x^4+1}\,dx = \int_0^1 \frac{x^3+x}{(x^2+1)(x^4+1)}\,dx =$$

$$= \int_0^1 \frac{x\cancel{(x^2+1)}}{\cancel{(x^2+1)}(x^4+1)}\,dx = \int_0^1 \frac{x}{x^4+1}\,dx = \int_0^1 \frac{x}{(x^2)^2+1}\,dx = *$$

$$\begin{array}{l} x^2 = t \\ 2x\,dx = dt \end{array} \Bigg|\; y = \frac{1}{2}\int_0^1 \frac{2x}{(x^2)^2+1}\,dx =$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+1} = \frac{1}{2}\operatorname{arctg} t \Big|_0^1$$

$$* = \frac{1}{2}\operatorname{arctg}(x^2)\Big|_0^1$$

$$= \frac{1}{2}\operatorname{arctg} 1 = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}.$$

c) $\displaystyle\lim_{x\to 1} \frac{1}{x-1}\cdot\int_1^x f(t)\,dt$

$$\int_1^x f(t)\,dt = \int_1^x \frac{t}{t^2+1}\,dt = \frac{1}{2}\ln t\Big|_1^x = \frac{\ln x}{2}$$

$$\lim_{x\to 1} \frac{\ln x}{2(x-1)} = \left(\frac{0}{0}\right) \overset{\text{L'Hospital}}{=} \lim_{x\to 1} \frac{1}{2x} = \frac{1}{2}.$$